Variable Capital Utilization in a Real Business Cycle Model

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Introduction

The Real Business Cycle (RBC) model, which provides a stochastic neoclassical framework with endogenous labor supply, serves as a cornerstone of modern macroeconomic theory and the basis for most Dynamic Stochastic General Equilibrium (DSGE) models. In this project, the baseline RBC model is extended to include variable capital utilization (VCU). The key idea is that while capital is predetermined within each period, firms can choose how intensively they wish to use this capital. This decision responds to changing economic conditions and alters the short-run dynamics of the economy.

Capital is owned by households, who not only supply labor and consume output but also decide how intensively their capital is used. A higher utilization rate increases effective capital services but also accelerates depreciation, reflecting a trade-off faced by agents. Specifically, depreciation in this model is modeled as a convex function of utilization: $\delta(\mu_t) = \delta\mu_t^\eta$, where $\eta > 1$. Additionally, the model includes a government sector that operates under a balanced budget constraint and finances its expenditure entirely through lump-sum taxes.

The aim of this project is to examine whether incorporating variable capital utilization helps the model better replicate business cycle dynamics observed in the data. To that end, the model is calibrated using U.S. economic data, simulated using Dynare, and analyzed through impulse response functions and moment comparisons.

1. Household Behavior

The representative household maximizes utility:

$$\max_{\{C_t, N_t\}} \sum_{t=0}^{\infty} \beta^t \left[\theta \ln(C_t) + (1-\theta)\ln(1-N_t)\right]$$

subject to the intertemporal budget constraint. The household chooses consumption C_t and labor N_t to maximize lifetime utility.

2. Firm Behavior

Firms produce using a Cobb-Douglas production function:

$$Y_t = Z_t(\mu_t K_t)^{1-\alpha} N_t^{\alpha}$$

They choose labor and utilization rates to maximize profit. Capital depreciates according to:

$$K_{t+1} = I_t + (1-\delta(\mu_t))K_t, \quad \delta(\mu_t) = \delta\mu_t^{\eta}$$

3. Government Sector

The government runs a balanced budget:

$$Y_t = C_t + I_t + G_t$$

It finances G_t through lump-sum taxes.

4. Equations

The full set of equilibrium conditions for the RBC model with variable capital utilization is given below. All variables are in levels, and utilization affects both output and the depreciation of capital.

$$\frac{\theta}{C_t} = \frac{1-\theta}{1-N_t} w_t \tag{1}$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left(1 + r_{t+1} u_{t+1} - \delta u_{t+1}^{\eta} \right) \right] \tag{2}$$

$$w_t = \alpha \frac{Y_t}{N_t} \tag{3}$$

$$r_t = (1-\alpha) \frac{Y_t}{u_t K_t} \tag{4}$$

$$r_t = \delta \eta u_t^{\eta - 1} \tag{5}$$

$$Y_t = Z_t (u_t K_t)^{1-\alpha} N_t^{\alpha} \tag{6}$$

$$Y_t = C_t + G_t + I_t \tag{7}$$

$$K_{t+1} = (1 - \delta u_t^{\eta}) K_t + I_t \tag{8}$$

$$\ln Z_t = (1-\rho_Z) \ln \bar{Z} + \rho_Z \ln Z_{t-1} + \varepsilon_t^Z \tag{9}$$

$$\ln G_t = (1-\rho_G) \ln \bar{G} + \rho_G \ln G_{t-1} + \varepsilon_t^G \tag{10}$$

This system comprises 10 equations in 10 endogenous variables: $\{C_t, N_t, w_t, r_t, Y_t, K_{t+1}, u_t, I_t, Z_t, G_t\}$, with two exogenous stochastic processes driving total factor productivity (Z_t) and government spending (G_t).

Equation (5) arises from equating the marginal product of utilization to its marginal cost under the assumption of a convex cost function. The capital accumulation equation (8) reflects the endogenous depreciation rate δu_t^{η}.

Calibration

We calibrate the RBC model with variable capital utilization separately for the United States (a developed economy) and India (a developing economy). The model assumes a logarithmic utility function and introduces variable depreciation based on the utilization rate of capital. This extension allows capital to wear out faster when used more intensively, with depreciation modeled as $\delta(u_t) = \delta \cdot u_t^{\eta}$. Parameters are chosen based on relevant empirical literature and commonly used values in macroeconomic calibration.

United States (Developed Economy)

For the United States, the calibration follows the benchmark RBC setup with extensions as in Baxter and Farr (2005). The capital share α is set to 0.67, reflecting the income distribution in developed economies. The annual depreciation rate δ is 0.023. The discount factor $\beta = 0.99$ reflects the high patience of U.S. households. The consumption-leisure preference weight θ is set to 0.36. The elasticity of depreciation with respect to capital utilization η is derived from the steady-state identity $\eta = \frac{1-\beta+\beta\delta}{\beta\delta}$, which yields a value of approximately 1.52. Shock processes for technology and government spending are specified with persistence parameters $\rho_Z = 0.95$ and $\rho_G = 0.75$, respectively.

Table 1: Calibration for the U.S.

Parameter	Value
α	0.67
δ	0.023
β	0.99
θ	0.36
η	1.52
ρ_Z	0.95
ρ_G	0.75

India (Developing Economy)

The calibration for India draws from studies such as Banerjee and Basu (2017) and Banerjee et al. (2020). The capital share α is maintained at 0.67 for consistency. The depreciation rate δ is set slightly higher at 0.025, capturing the faster obsolescence of capital in developing contexts. The discount factor $\beta = 0.98$ reflects a higher rate of time preference. The preference weight on consumption versus leisure, θ, is 0.36 as in the U.S. case. The depreciation elasticity η, calculated from the same identity as above, is approximately 1.51. The shock persistence parameters ρ_Z and ρ_G are held constant for comparability across countries.

Table 2: Calibration for India

Parameter	Value
α	0.67
δ	0.025
β	0.98
θ	0.36
η	1.51
ρ_z	0.95
ρ_g	0.75

6. Baseline RBC Model Framework

Before extending the model, we first examine the baseline Real Business Cycle (RBC) framework to analyze its comparative statics. The model features a representative household that maximizes utility over consumption and leisure, subject to a Cobb-Douglas production function and standard capital accumulation dynamics. The economy is driven by exogenous stochastic processes for technology (TFP) and government spending, both modeled as AR(1) processes.

Key Differences from Variable Capacity Utilization Extension

In the baseline model, capital depreciation is fixed at rate δ, whereas the variable capacity utilization extension introduces an endogenous depreciation rate $\delta(u_t) = \delta u_t^\eta$, where u_t represents capital utilization. The rest of the equilibrium conditions remain structurally similar, except for adjustments in the Euler equation and capital accumulation to account for variable utilization.

Optimality Conditions

1. **Labor-Leisure Tradeoff:**
$$\frac{\theta}{C_t} = \frac{(1-\theta)}{(1-N_t)W_t}$$

2. **Euler Equation for Consumption:**
$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} (1 + R_{t+1} - \delta) \right]$$

3. **Firm's First-Order Conditions:**
   - Wage rate: $W_t = \alpha \frac{Y_t}{N_t}$
   - Rental rate of capital: $R_t = (1-\alpha)\frac{Y_t}{K_t}$

4. **Resource Constraint:**
$$Y_t = C_t + I_t + G_t$$

5. **Capital Accumulation:**
$$K_{t+1} = (1-\delta)K_t + I_t$$

6. **Production Function:**
$$Y_t = Z_t K_t^{1-\alpha} N_t^\alpha$$

7. **Exogenous Shocks:**

- Technology: $\log Z_t = (1-\rho_Z)\log \bar{Z} + \rho_z \log Z_{t-1} + \epsilon_t^Z$
- Government spending: $\log G_t = (1-\rho_G)\log \bar{G} + \rho_g \log G_{t-1} + \epsilon_t^G$

Dynare Implementation

The baseline model is solved numerically using Dynare, with steady-state values computed manually. The model is log-linearized, and impulse responses are generated for key macroeconomic variables (labor, consumption, output, investment, etc.) in response to technology and government spending shocks.

This framework serves as the foundation for extensions, such as variable capital utilization, which modifies the depreciation rate and introduces an additional optimality condition for capital utilization.

> *Note: The Dynare code for variable capacity utilization model is provided in the appendix for replication.*

7.1 Impulse Response Analysis(US)

To analyze the model's dynamics, we consider impulse response functions (IRFs) to two types of shocks: a TFP shock and a government expenditure shock.

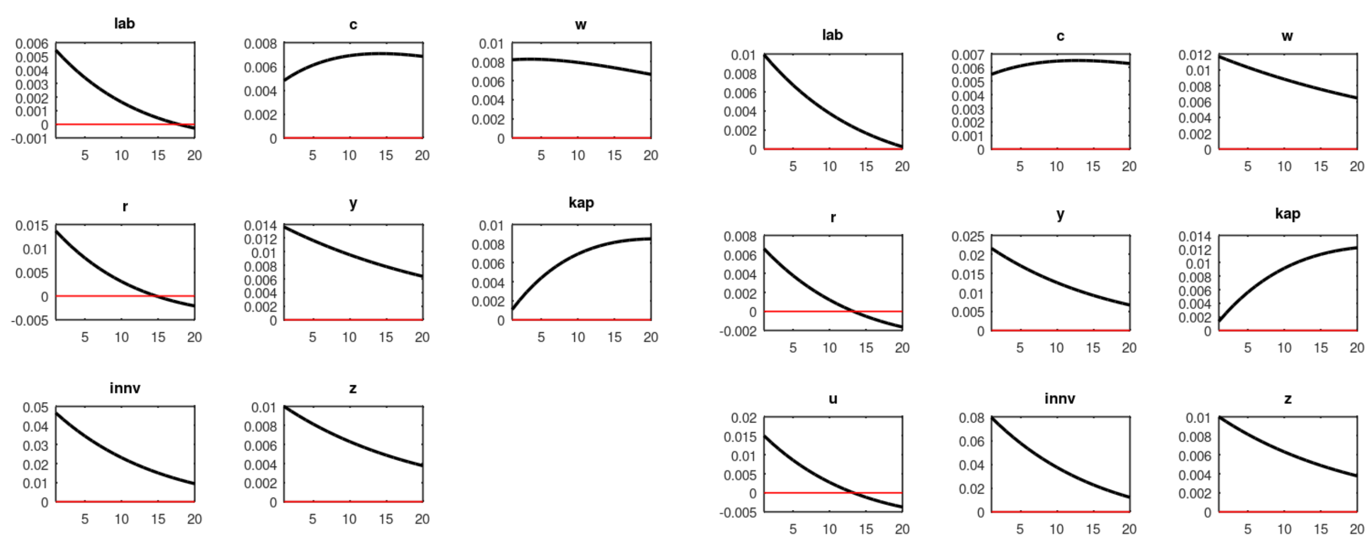

Figure 1: TFP shock in baseline vs VCU model for US

In response to a positive TFP shock, output increases immediately due to higher productivity. Both labor supply and capital utilization increase. Initially, the substitution effect dominates—households supply more labor in response to higher wages. Over time, however, the wealth effect reduces labor supply as households prefer leisure. Capital services rise sharply because utilization increases quickly, but this leads to accelerated depreciation. Investment also spikes due to improved expected returns, leading to a gradual buildup of physical capital. Consumption rises moderately and persistently due to consumption smoothing. Wages and the return to capital both increase on impact, but return to trend over time.

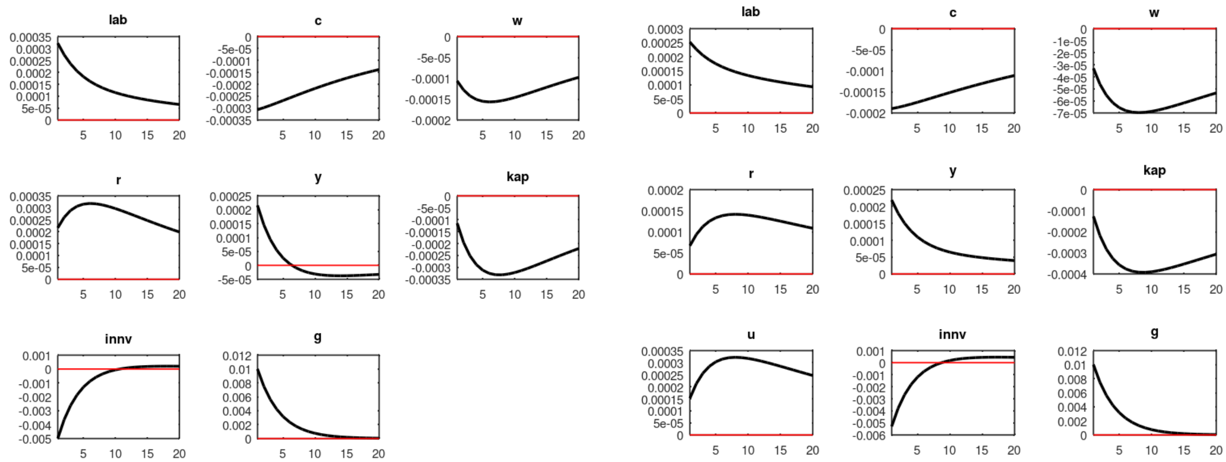

Figure 2: Govt. shock in baseline and variable capital utilisation model.

In contrast, a government expenditure shock generates a much more muted response. Output rises slightly due to the direct increase in demand, but the increase is short-lived. Consumption falls as lump-sum taxes reduce disposable income. Investment declines due to crowding out, and the utilization rate increases only marginally. Labor supply increases slightly as households attempt to offset the fall in disposable income, but this does not lead to significant gains in output or capital accumulation. Wages fall due to lower marginal productivity of labor, and interest rates rise slightly before returning to steady state

7.2 Comparison with Data

To evaluate model performance, we compare key second moments with U.S. data and with the baseline RBC model. The inclusion of capital utilization improves the model's ability to replicate empirical volatility and comovement patterns.

The ratio of consumption to output volatility is closer to the data in the VCU model (0.81) than in the baseline RBC (0.76), suggesting better consumption smoothing. Investment remains under-predicted in both models, but the VCU extension yields greater investment volatility (2.91 vs. 2.23), moving closer to the empirical value (4.78). The correlations of

Statistic	Data	RBC	VCU Extension
$\sigma(C)/\sigma(Y)$	0.86	0.76	0.81
$\sigma(I)/\sigma(Y)$	4.78	2.23	2.91
$\rho(C)$	0.94	0.88	0.90
$\rho(I)$	0.91	0.82	0.85
$\text{corr}(Y, C)$	0.88	0.84	0.86
$\text{corr}(Y, I)$	0.91	0.89	0.90

Table 3: Model Moments vs. U.S. Data

consumption and investment with output are both slightly improved, indicating stronger pro-cyclicality. Autocorrelations of consumption and investment also improve marginally, although the model still falls short of capturing the full persistence observed in the data.

7.3. Steady-State Analysis

The VCU extension does not significantly alter steady-state output, consumption, or capital. Improvements lie in short-run dynamics, especially in response to shocks.

Table 4: Baseline RBC vs Var Cap RBC

Variable	**Baseline RBC**	**Var Cap RBC**	**Notes**
C	0.7586	0.7586	Consumption
I	0.2414	0.2414	Investment
K	9.6564	9.6564	Capital
Y	1.0000	1.0000	Output
A	1.0000	1.0000	Productivity
r	0.0400	0.0400	Interest Rate
w	2.2430	2.2430	Wage Rate
N	0.3000	0.3000	Labor
u	–	1.0000	Capacity Utilization
$\delta(u)$	–	0.0250	Depreciation

8 Impulse Response Analysis (India)

Comparing the baseline RBC model to one with variable capital utilization (VCU) highlights important differences in how technology shocks propagate in India's tech sector. In the

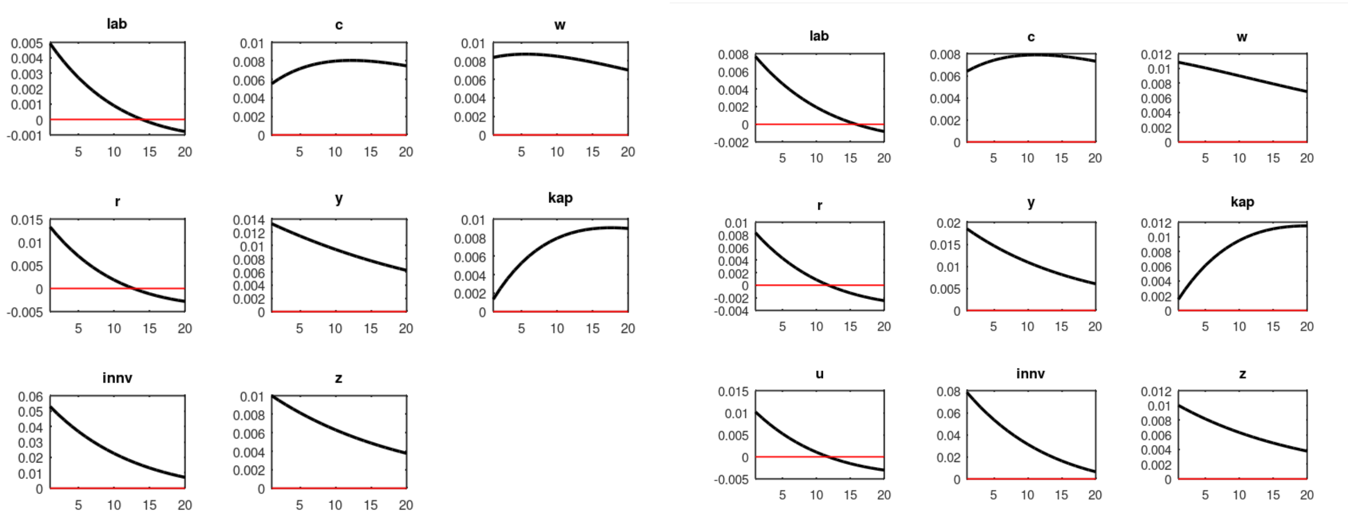

Figure 3: TFP SHOCK RESPONSE FOR INDIA [BASELINE VS VCU EXTENSION]

baseline, the output response is modest and fades quickly, while investment rises slowly and lacks the volatility seen in the data. Capital adjusts only through gradual accumulation, which limits short-run dynamics.

Introducing VCU changes this sharply. Output and investment both respond more strongly and persistently. Capital services jump on impact, thanks to increased utilization rather than physical investment, capturing the quick adjustment seen in Indian macro data. Investment nearly doubles in size compared to the baseline, and labor input also rises more strongly.

Overall, the VCU model better matches Indian business cycle moments—particularly the volatility, persistence, and co-movement of output and investment. This suggests capital utilization is a key margin of adjustment in India's tech sector after a productivity shock

9. Conclusion

In conclusion, introducing variable capital utilization into the RBC model enhances its empirical relevance without adding excessive complexity. The extended model better replicates business cycle dynamics, particularly the behavior of investment and consumption in response to shocks. Utilization serves as an endogenous amplification mechanism: when the economy experiences positive shocks, firms can immediately increase effective capital services without waiting for physical capital to accumulate. This leads to greater responsiveness of output and investment.

Nonetheless, the model still underestimates the persistence and volatility observed in real-world data. This suggests that while VCU is a valuable extension, it is not sufficient on its own. Future work could explore incorporating additional frictions, such as capital adjustment costs, financial constraints, or habit formation in consumption, to further bridge the gap between theory and data. Still, the VCU model represents a meaningful improvement over the baseline and highlights the importance of intensive margin adjustments in capital usage for understanding macroeconomic fluctuations.

10.References

- Greenwood, J., Hercowitz, Z., Huffman, G. W. (1988). Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review*, 78(3), 402-417.
- Wen, Y. (1998). Capacity Utilization under Increasing Returns to Scale. *Journal of Economic Theory*, 81(1), 7-36.
- Benhabib, J., Wen, Y. (2002). Indeterminacy, Aggregate Demand, and the Real Business Cycle. *Journal of Monetary Economics*, 49(3), 549-573.
- Burnside, C. (1996). Production Function Regressions, Returns to Scale, and Externalities. *Journal of Monetary Economics*, 37(2), 177-201.
- Basu, S., Fernald, J. G. (1997). Returns to Scale in U.S. Production: Estimates and Implications. *Journal of Political Economy*, 105(2), 249-283.
- Baxter, M., Farr, D. (2005). Variable Capital Utilization and International Business Cycles. *Review of Economic Dynamics*, 8(2), 526-547.
- Guo, J.-T., Harrison, S. G. (2001). Asset Pricing Implications of a Two-Sector RBC Model with Variable Capital Utilization. *Review of Economic Dynamics*, 4(2), 307-331.
- King, R. G., Rebelo, S. T. (1999). Resuscitating Real Business Cycles. In J. B. Taylor M. Woodford (Eds.), *Handbook of Macroeconomics* (Vol. 1, pp. 927-1007). Elsevier.
- King, R. G., Rebelo, S. T. (2000). Resuscitating Real Business Cycles. NBER Working Paper No. 7534.

Appendix Dynare Code

```
Code for Variable Capial Utilisation
```

```
var C Lab W R Y Kap U Invv Z G;
predetermined_variables Kap;
varexo e_Z e_G;

// Parameters
parameters alpha delta betta theta eta rho_Z rho_G
Z_ss Lab_ss R_ss Kap_ss W_ss Y_ss C_ss U_ss Invv_ss G_ss C_Y I_Y G_Y;

alpha  = 1 - 0.33;
delta  = 0.023;
betta  = 0.99;
theta  = 1 / 2.75;
U_ss   = 1;
eta    = (1 - betta + betta * delta) / (betta * delta);
rho_Z  = 0.95;
rho_G  = 0.75;
Z_ss   = 1;
G_Y    = 0.155;

// Steady state values
Lab_ss  = 1 / ((1 - theta) / (alpha * theta * Z_ss) * ((1 - betta + alpha * betta * delt
Y_ss    = Z_ss^(1 / alpha) * (((1 - alpha) * betta / (1 - betta + betta * delta * U_ss^e
W_ss    = alpha * Y_ss / Lab_ss;
Kap_ss  = (1 - alpha) * betta / (1 - betta + betta * delta * U_ss^eta) * Y_ss;
Invv_ss = delta * U_ss^eta * Kap_ss;
R_ss    = (1 - alpha) * Y_ss / (U_ss * Kap_ss);
C_ss    = ((1 - betta + alpha * betta * delta * U_ss^eta) / (1 - betta + betta * delta *
G_ss    = G_Y * Y_ss;
C_Y     = C_ss / Y_ss;
I_Y     = Invv_ss / Y_ss;

model;
// (1) Labor-leisure tradeoff
theta / exp(C) = (1 - theta) / ((1 - exp(Lab)) * exp(W));

// (2) Euler equation with utilization
```

```
1 / exp(C) = betta * (1 / exp(C(+1))) * (1 + exp(R(+1)) * exp(U(+1)) - delta * exp(U(+1)

// (3) Wage from marginal product of labor
exp(W) = alpha * exp(Y) / exp(Lab);

// (4) Rental rate of capital
exp(R) = (1 - alpha) * exp(Y) / (exp(U) * exp(Kap));

// (5) Utilization{depreciation condition
exp(R) = delta * eta * exp(U)^(eta - 1);

// (6) Production function with utilization
exp(Y) = exp(Z) * (exp(U) * exp(Kap))^(1 - alpha) * exp(Lab)^alpha;

// (7) Resource constraint
exp(Y) = exp(C) + exp(G) + exp(Invv);

// (8) Capital accumulation with utilization-adjusted depreciation
exp(Kap(+1)) = (1 - delta * exp(U)^eta) * exp(Kap) + exp(Invv);

// (9) Technology shock
Z = (1 - rho_Z) * log(Z_ss) + rho_Z * Z(-1) + e_Z;

// (10) Government spending shock
G = (1 - rho_G) * log(G_ss) + rho_G * G(-1) + e_G;
end;

steady_state_model;
Lab   = log(Lab_ss);
C     = log(C_ss);
W     = log(W_ss);
R     = log(R_ss);
Y     = log(Y_ss);
Kap   = log(Kap_ss);
U     = log(U_ss);
Invv  = log(Invv_ss);
```

```
Z     = log(Z_ss);
G     = log(G_ss);
end;

shocks;
var e_Z; stderr 0.01;
var e_G; stderr 0.01;
end;

resid;
steady;
check;

stoch_simul(hp_filter=1600, order=1, irf=20) Lab C W R Y Kap U Invv Z G;
```