Testing the Capital Asset Pricing Model (CAPM)

Group 6 – Stock Returns & Risk Factors

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Case Study Question

Scenario

A financial analyst tests the Capital Asset Pricing Model (CAPM) for a mid-cap firm listed on the stock exchange using monthly data on:

- Firm's stock returns
- Market index returns
- Risk-free Treasury bill rates

Objective

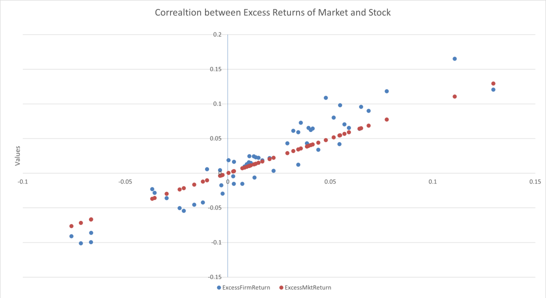
To test whether the firm's excess returns are explained by market excess returns and to estimate its risk (beta).To check whether CAPM assumptions hold for this dataset

The Great CAPM Debate

- **Sharpe (1964), Lintner (1965), Mossin (1966)** formalize CAPM, proposing that expected returns depend solely on market beta.
- **Black, Jensen & Scholes (1972)** provide early empirical support but note beta instability across periods.
- **Fama & MacBeth (1973)** develop a two-pass regression test; find a positive beta–return relation, though weaker than theory predicts.
- **Banz (1981)** documents the size effect; small firms earn excess returns unexplained by beta.
- **Fama & French (1992)** show beta alone has limited explanatory power and introduce size and value factors, challenging traditional CAPM.
- **Jagannathan & Wang (1996)** argue that CAPM performs better when accounting for human capital and time-varying betas.
- **Consensus**: CAPM remains a foundational benchmark in finance, but multi-factor models generally provide superior empirical performance.

Summary statistics

Correlation Plot



Summary Statistics

	FirmReturn	MarketReturn	RiskFree	ExcessFirmReturn	ExcessMktReturn
Mean	0.02372926	0.019292982	0.00005763	0.021273497	0.017287219
Standard Error	0.057268945	0.041351585	0.00003847	0.057126817	0.041292697

Table: Summary statistics of returns and excess returns

The CAPM Model

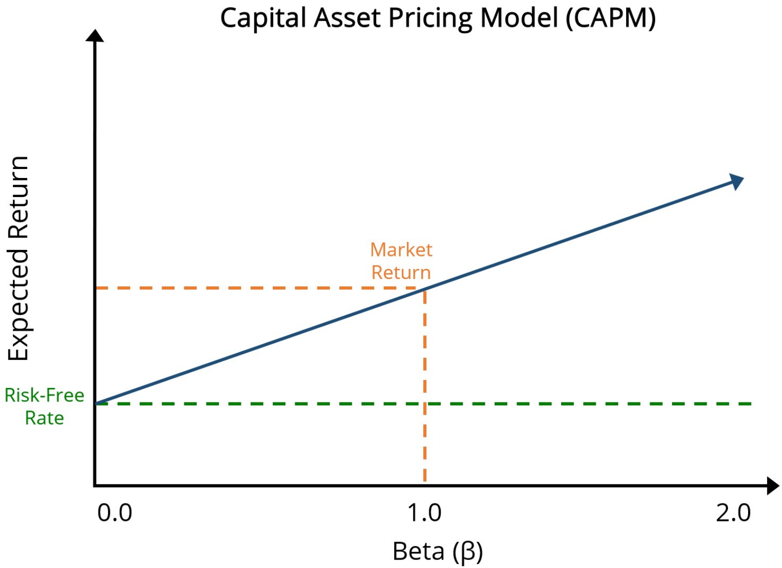
A model that assumes there is a linear relationship between risk and return, and return is only explained by the riskiness relative to market.

Equation

$$E(R_i - R_f) = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i$$

- R_i: Return on firm i
- R_m: Market return
- R_f: Risk-free rate
- α_i: Abnormal return (Jensen's alpha)
- β_i: Systematic risk of the firm

CAPM


Capital Asset Pricing Model (CAPM)
Expected Return
Market
Return
Risk-Free
Rate
0.0
1.0
2.0
Beta (β)

Assumptions of CAPM

- Markets are **efficient** : all available information is reflected in security prices.
- Investors are **rational** and risk-averse, making decisions based on expected return and variance.
- All investors share a **single investment horizon** (typically one period).
- Investors have **homogeneous expectations** about returns, risks, and correlations.
- There are **no taxes or transaction costs**; trading is frictionless.
- Investors can **borrow and lend freely at the risk-free rate**.
- Assets are **infinitely divisible**, allowing fractional holdings.
- Each investor is a **price taker** and cannot influence market prices.
- **Short selling is allowed** without restrictions.
- **All assets are marketable** and can be traded in the market.

Detour: Two–Step CAPM Estimation

Step 1: Estimate Beta

- Run the regression:

$$R_i = \alpha + \beta R_m + u_t$$

- Obtain residuals:

$$\hat{u}_t = R_i - \widehat{\alpha} - \widehat{\beta} R_m$$

Step 2: Test Model Assumptions

- Use this $\hat{u}_t$ as an proxy for other variables like firm size, human capital , etc.
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$$E(R_i - R_f) = \alpha_i + \beta(R_m - R_f) + \gamma \hat{u}_t + \epsilon_t$$

Detour: Two–Step CAPM Estimation Results

Regression Output (Step 2):
Dependent variable: Excess Firm Return

Variable	**Coef.**	**Std. Err.**	**t**	**p-value**
Excess Market Return	1.329272	0.0663162	20.04	0.000
Residual Sq. ($\hat{u}^2$)	-5.123293	5.974942	-0.86	0.395
Constant	0.000493	0.0032228	0.02	0.988

Table: Step 2 Regression: CAPM Augmented with Residual Variance Proxy

- Non–market (idiosyncratic) risk does not significantly explain excess returns in our sample — consistent with CAPM.

Regression Model:

$$(R_i - R_f) = \alpha + \beta(R_m - R_f) + \varepsilon_t$$

Table: CAPM Regression Results

Parameter	**Estimate**
Intercept (α)	-0.0011 ($p = 0.70$)
Beta (β)	1.3165 ($p < 0.001$)
R^2	0.896
F-statistic	449.49 ($p < 0.001$)
Std. Error	0.0188
Observations	55

Interpretation:

- $\beta = 1.3165 > 1 \Rightarrow$ the firm's stock is **more volatile** than the market.
- α is insignificant, implying CAPM holds reasonably well.

CAPM assumption holds?

Autocorrelation:

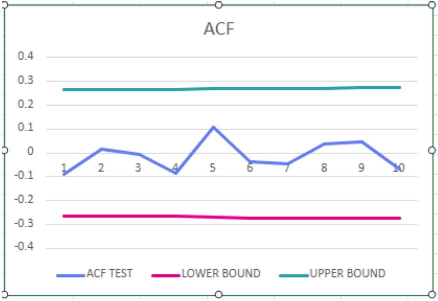
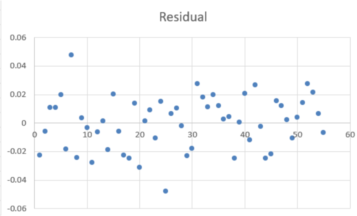


Figure: Diagnostic Plots – Autocorrelation

- No Autocorrelation was found.

Heteroskedasticity

Residual plot



Null Hypothesis: Errors are homoskedastic.

Test	LM Statistic	p-value
White's Test	10.751	0.005
Breusch–Pagan Test	6.232	0.013

Both the Breusch–Pagan (BP) test and White's test yield p-values less than 0.05. Therefore, we fail to accept the null hypothesis of homoskedasticity.

Robust Regression (Correcting for Heteroskedasticity)

- **Model Specification:**

$$(R_i - R_f) = \alpha + \beta(R_m - R_f) + \varepsilon_t$$

- Breusch–Pagan test detected heteroskedasticity ($p = 0.017 < 0.05$), so robust standard errors were used.

Table: Robust Regression Results

Variable	**Coef.**	**Std. Err.**	**t**	**P$>|t|$**	**[95% Conf. Interval]**
Excess Market Return	**1.3097**	0.0782	16.74	0.000	[1.1529, 1.4666]
Constant (α)	-0.0014	0.0024	-0.57	0.574	[-0.0062, 0.0035]
N = 55 observations	*Adjusted R^2 = 0.893*				

- **Interpretation:**
  - $\beta = 1.31 > 1 \rightarrow$ firm's stock is **more volatile** than the market.
  - α remains insignificant ($p = 0.57$) $\rightarrow$ no abnormal return; CAPM holds.
  - Robust regression confirms results are statistically reliable.

Dickey–Fuller Test Results

Null Hypothesis: Series is stationary.

- The p-values for all variables are less than 0.05, so we fail to reject the null hypothesis.
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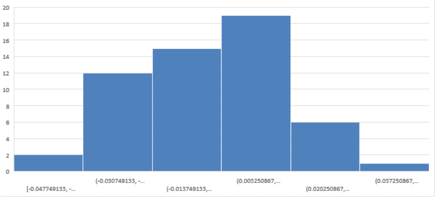
Variable	**t-Statistic**	**Critical Value (1%)**	**Conclusion**
Excess Firm Retun	-6.729^{***}	-3.574	Stationary
Excess Market Retun	-7.075^{***}	-3.574	Stationary

Table: Dickey–Fuller Test: All variables are stationary (I(0))

Note: *** indicates significance at the 1% level.

Normality

Normality of residuals



J-B Test for Normality

Test	JB Statistic	p-value
Jarque–Bera Normality Test	0.20	0.90

Table: Jarque–Bera Test: Residuals follow normal distribution

Implications and Conclusion

Key Findings:

- A/c to the CAPM, the only relevant measure of a stock's risk is the systematic risk β.
- α is statistically insignificant ($p = 0.57$) — no abnormal returns beyond what CAPM predicts.
- $\beta = 1.31 > 1$ — The firm's stock is **more volatile** than the market, implying higher risk of earning excess profit and also losing higher amount of money during market downturn.
- Model fit is strong ($R^2 = 0.89$), indicating that excess market return explains most of the variation in excess firm return.
- Robust regression confirms that results remain stable after correcting for heteroskedasticity (common for financial data).

Interpretation:

- CAPM holds reasonably well for the firm — returns are largely driven by market movements.
- The firm's stock has more volatility compared to market, rewarding investors with high risk tolerance.

Managerial Implications:

- Investors should expect higher risk and proportionally higher returns from this stock.
- Portfolio managers can use the estimated β to assess diversification and market exposure.

Managerial Implication: Using CAPM for Decision Making

Example: Calculating Cost of Equity Using CAPM

$$E(R_i) = R_f + \beta(R_m - R_f)$$

$$\text{If } R_f = 6\%, \quad (R_m - R_f) = 8\%, \quad \beta = 1.3$$

$$E(R_i) = 6 + 1.3 \times 8 = 16.4\%$$

Interpretation: The firm requires a minimum return of **16.4%** to justify an investment. This value is used as the **cost of equity**.

Key Managerial Insight

- CAPM converts market risk into expected return.
- Managers use this expected return as a hurdle rate for projects.
- Helps assess whether a stock or project is **fairly priced and worth investing in**.